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Replies to Readers' Questions

The Geometric Coefficient of Variation
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BACKGROUND: In statistics, the Arithmetic Coefficient of Variation—a relative and dimensionless measure of dispersion—is equal to the standard deviation divided by the mean.

QUESTION: In "Deceptive Means," you state that the geometric standard deviation is a "factor, not a quantity." This would seem to be true for the geometric mean as well. Thus, it would seem valid to form a dimensionless "geometric coefficient of variation" as the geometric standard deviation over the geometric mean. But perhaps I'm overlooking something. — Shaun

REPLY: Hi, Shaun. I've not had a real-world problem where the geometric coefficient of variation was of practical use to me, so until now I haven't thought much about it. Nevertheless, your question started the wheels turning. Here's how I explain it to myself. (I've never seen a textbook discussion of it, though surely there must be one out there.)

In the case of the *arithmetic* coefficient of variation, the mathematical operation that relates the standard deviation to the mean is *division*. In the case of the *geometric* coefficient of variation, the mathematical operation that relates the standard deviation to the mean is *root extraction*.

To see why this is so, let's consider the "food chain" of mathematical operations. On the bottom level are *addition* and *subtraction*. At the next level are, respectively, *multiplication* and *division*. And at the level above that are, respectively, *elevation to a power* and *extraction of a root*.

In an *arithmetic* framework, we *generate* the mean using *division*, and we *apply* the standard deviation to it using *addition* or *subtraction*. In a *geometric* framework, we *generate* the mean using *root extraction*, and we *apply* the standard deviation to it using *multiplication* or *division*.

So the operation that generates the geometric mean (i.e., root extraction) is *one level higher* on the mathematical food chain than the operation that generates the arithmetic mean (i.e., division). And the operation that applies the geometric standard deviation to the geometric mean (i.e., multiplication or division) is *one level higher* on the mathematical food chain than the operation that applies the arithmetic standard deviation to the arithmetic mean (i.e., addition or subtraction).

The same principle holds true for the geometric coefficient of variation. The operation that *relates* the geometric standard deviation to the geometric mean is root extraction—*one level higher* than division (the operation that relates the arithmetic standard deviation to the arithmetic mean in the arithmetic coefficient of variation).

Let’s summarize. To obtain the *arithmetic* coefficient of variation, we divide the arithmetic standard deviation by the arithmetic mean. To obtain the *geometric* coefficient of variation, we reduce the geometric standard deviation to the power of the reciprocal of the geometric mean. (Recall that an exponent *n* raises a number to the *n*th power, while its reciprocal exponent, *1/n,* takes the *n*th root of the number—which is what we're after.)

A calculation of the geometric coefficient of variation looks like this. Assume a geometric standard deviation of 1.02 and a geometric mean of 1.08. The geometric coefficient of variation = 1.02^(1/1.08) = 1.018504898.

We can obtain the same result using logarithms. Divide the log of the geometric standard deviation by the geometric mean (NOT the log of the geometric mean); then take the antilog of the result. Here's the same example using base 10 logarithms: (LOG of geometric standard deviation)/geometric mean = (LOG 1.02)/1.08 = .008600171/1.08 = .007963122. ANTILOG .007963122 = 1.018504898.