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Deceptive Means:
The Average Guy's Guide to EPS Growth Rates

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Crystal balls are in short supply, especially in the stock market. Yet every investor wants to know the growth rate of future earnings per share (EPS). While the norms of a company's past EPS growth are no sure indicator of the future, they do lend perspective.

If you decide to calculate these norms, it is important to do it correctly. Otherwise, bogus results will warp your frame of reference. Here are the procedures to use and the pitfalls to avoid when you compute growth-rate norms.

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How to begin. Collect a company's annual EPS data for an extended time—say, seven to ten years. Arrange the EPS in chronological order. All EPS must be positive, as in this ten-year sequence from a large southeastern bank.

$2.25, $2.47, $2.76, $3.13, $3.04, $3.50, $4.30, $4.70, $4.66, $4.73

Make ratios of the sequence's adjacent EPS, putting the earlier EPS in the ratio's denominator and the later EPS in its numerator, like this.

$2.47/$2.25
$2.76/$2.47
$3.13/$2.76
$3.04/$3.13
$3.50/$3.04
$4.30/$3.50
$4.70/$4.30
$4.66/$4.70
$4.73/$4.66

Calculate the values of the ratios, dividing each ratio's denominator into its numerator. (These values are rounded.)

$2.47/$2.25 = 1.0978
$2.76/$2.47 = 1.1174
$3.13/$2.76 = 1.1341
$3.04/$3.13 = 0.9712
$3.50/$3.04 = 1.1513
$4.30/$3.50 = 1.2286
$4.70/$4.30 = 1.0930
$4.66/$4.70 = 0.9915
$4.73/$4.66 = 1.0150

The ratio values are *annual growth factors* for the EPS. A growth factor always equals *1+g*. The number *1* stands for 100% of the earlier EPS. And *g* is the EPS growth rate—it tells us the proportion of the earlier EPS (the denominator) by which the later EPS (the numerator) increased or decreased.

Since the growth factor is *1+g*, the growth rate *g* is *the growth factor minus 1*. Here are two growth-rate examples from the above data.

- *g*, the growth rate = [growth factor - 1] = [($3.13/$2.76) – 1] = 1.1341 – 1 = .1341, which is positive EPS growth of +13.41%.
- *g*, the growth rate = [growth factor - 1] = [($3.04/$3.13 – 1] = 0.9712 – 1 = -.0288, which is negative EPS growth of -2.88%

Once we have calculated the EPS growth factors, we can examine their year-to-year volatility—an important thing to look into. But more significantly, we can determine their norms; specifically, (1) the growth factors' *mean* (i.e., their average value) and (2) the growth factors' *standard deviation* (i.e., the degree to which they are scattered around their mean).

A pitfall: the arithmetic mean and standard deviation. There are different kinds of means. The most familiar is the *arithmetic* mean. To calculate the arithmetic mean of *n* growth factors, we add them and divide their sum by *n*, as follows:

(1.0978+1.1174+1.1341+. 9712+1.1513+1.2286+1.0930+. 9915+1.0150)/9
= 9.7999/9
= 1.0889, the arithmetic mean of the growth factors—*the quotient of* a nine-number sum divided by nine

The scatter of growth-factor values around their arithmetic mean is measured by the *arithmetic standard deviation*. The labor-intensive computation of the arithmetic standard deviation is programmed into spreadsheets and inexpensive scientific calculators. Most statistics books explain the underlying math.

The arithmetic standard deviation is a *quantity*. *Multiples* of that quantity are *added* to (or *subtracted* from) the arithmetic mean to determine the set of values that lie within a given range of dispersion. To determine the value that lies *n* standard deviations from the mean, we must multiply the arithmetic standard deviation by *n*.

The above growth factors have an arithmetic standard deviation of .083. The growth factor that lies, say, two standard deviations *above* the mean is 1.2549, which equals the mean *plus* two standard deviations: 1.0889 + 2(.083) = 1.2549. The value that lies two standard deviations *below* the mean is .9229, which equals the mean *minus* two standard deviations: 1.0889 – 2(.083) = .9229.

Later, we'll use these procedures as stepping-stones to our goal—but only as that. In and of themselves, the arithmetic mean and standard deviation are inappropriate measures of growth-factor norms.

The geometric mean and standard deviation. The appropriate measures of growth-factor norms are the less-familiar geometric mean and standard deviation.

To calculate the geometric mean of *n* growth factors, we multiply the numbers together and take the n^{th} root of their product, as shown here.

$[(1.0978)(1.1174)(1.1341)(.9712)(1.1513)(1.2286)(1.0930)(.9915)(1.0150)]^{1/9}$
$= 2.1022^{1/9}$
= 1.0861, the geometric mean of the growth factors—*the ninth root of* a nine-factor product

We can also find the geometric mean using a shortcut: Divide the final year's earnings (EPS_n) by the first year's earnings (EPS_0), and take the n^{th} root of the result. In our banking example, the final EPS is $4.73, the first EPS is $2.25, and *n* is nine because the ten EPS span nine year-long periods. In general, the number of growth factors is one less than the number of EPS.

$(EPS_n/EPS_0)^{1/n}$
$= (\$4.73/\$2.25)^{1/9}$
$=(2.1022)^{1/9}$
= 1.0861, the geometric mean of the growth factors

The scatter of growth-factor values around their geometric mean is measured by the *geometric standard deviation*. The geometric standard deviation can be found using a spreadsheet or a scientific calculator, but first the growth factors must be converted into logarithms (as explained below).

In contrast to the arithmetic standard deviation, the geometric standard deviation is not a *quantity*; it is a *factor*. *Powers* of the geometric standard deviation are *multiplied* by (or *divided* into) the geometric mean to determine the set of values that lie within a given range of dispersion. To determine the value that lies *n* geometric standard deviations from the mean, we must raise the geometric standard deviation to the n^{th} power.

The above growth factors have a geometric standard deviation of 1.0795. The growth factor that lies, say, two standard deviations *above* the mean is 1.2657—which equals the mean *times* the second power of the standard deviation: $(1.0861)(1.0795^2) = 1.2657$. And the value that lies two standard deviations *below* the mean is .9320—which equals the mean *divided by* the second power of the standard deviation: $1.0861/(1.0795^2) = .9320$.

A comparison of arithmetic and geometric norms. In the above examples, there is little difference between the growth factors' arithmetic mean (1.0889) and geometric mean (1.0861). Similarly, there is little difference in the values that lie two standard deviations above those means—1.2549 (arithmetic) and 1.2657 (geometric). And little difference in the values that lie two standard below those means—.9229 (arithmetic) and .9320 (geometric). So why even bother with the geometric mean and standard deviation?

Because analytical tools must fit the kind of data they are examining—if they don't, they can generate seriously distorted results.

The arithmetic mean and arithmetic standard deviation are sum-based values. As such, they are appropriate for additive processes. But earnings growth is not additive; it is multiplicative. The appropriate measures for EPS growth factors are the geometric mean and geometric standard deviation, which are product-based values.

To drive this point home, we'll apply both the arithmetic mean and the geometric mean to our bank's EPS sequence and examine the results. Recall that a ten-EPS sequence has nine growth factors. The first EPS times the nine growth factors equals the final EPS.

EPS_0[the *n* growth factors] = EPS_n
$2.25[(1.0978)(1.1174)(1.1341)(.9712)(1.1513)(1.2286)(1.0930)(.9915)(1.0150)]
= $2.25[2.1022]
= $4.73, the final EPS

It follows that the first EPS times nine *mean-growth factors* should also equal the final EPS. But when we use the arithmetic mean, 1.0889, this doesn't work out. The result is not the final EPS of $4.73, but $4.84, which is eleven cents higher.

$2.25[(1.0889)(1.0889)(1.0889)(1.0889)(1.0889)(1.0889)(1.0889)(1.0889)(1.0889)]
= $2.25[1.0889^9]
= $2.25[2.1522]
= $4.84
≠ $4.73, the final EPS

It is the geometric mean, 1.0861, that produces the final *EPS*, $4.73.

$2.25[(1.0861)(1.0861)(1.0861)(1.0861)(1.0861)(1.0861)(1.0861)(1.0861)(1.0861)]
= $2.25[1.0861^9]
= $2.25[2.1029]
= $4.73, the final EPS
(Because our geometric mean is rounded, the value of 1.0861^9 is slightly higher than the product of the nine annual growth factors.)

Average dangers. We now turn to two EPS sequences whose arithmetic and geometric means have markedly different values.

High or higher? The first sequence comes from a manufacturer of medical devices. Here are its EPS in chronological order.

$0.32, $0.45, $0.55, $0.48, $0.40, $0.90, $0.85, $0.80, $1.30, $1.60

And here are the resulting growth factors.

$0.45/$0.32 = 1.4063
$0.55/$0.45 = 1.2222
$0.48/$0.55 = 0.8727
$0.40/$0.48 = 0.8333
$0.90/$0.40 = 2.2500
$0.85/$0.90 = 0.9444
$0.80/$0.85 = 0.9412
$1.30/$0.80 = 1.6250
$1.60/$1.30 = 1.2308

The arithmetic mean of these factors is 1.2584, implying that EPS grew at an average annual rate of 25.84%. Had that been so, the first EPS, $0.32, would have grown into $2.53 in the last year—which, of course, it did not.

$EPS_0 (1+g)^n = EPS_n$
$0.32(1.2584^9) = $2.53
≠ $1.60, the final EPS

The true mean is 1.1958, the geometric mean. The first EPS, growing at an annual rate of 19.58%, does indeed become the final EPS, $1.60.

$EPS_0 (1+g)^n = EPS_n$
$0.32(1.1958^9) = $1.60, the final EPS

If you accept the arithmetic mean for the norm, you will overstate the average annual growth rate by 6.26%.

Growing or shrinking? We end this section with the EPS sequence of a global communications company, where there is a dramatic difference between the two means. Here are its EPS in chronological order.

$1.47, $1.84, $2.01, $1.57, $1.87, $2.66, $3.95, $0.22, $1.67, $1.27

And here are the resulting growth factors.

$1.84/$1.47 = 1.2517
$2.01/$1.84 = 1.0924
$1.57/$2.01 = 0.7811
$1.87/$1.57 = 1.1911
$2.66/$1.87 = 1.4225
$3.95/$2.66 = 1.4850
$0.22/$3.95 = 0.0557
$1.67/$0.22 = 7.5909
$1.27/$1.67 = 0.7605

The arithmetic mean of these factors is 1.7368, implying that EPS grew at an average annual rate of 73.68%. Had that been so, the first EPS, $1.47, would have grown into a whopping $211.38 in the final year!

$EPS_0\ (1+g)^n = EPS_n$
$\$1.47(1.7368^9) = \$211.38$
$\neq$ $1.27, the final EPS

The true mean is the geometric mean, 0.9839, which reveals an average annual *decrease* in EPS of -1.61%. The first EPS growing (negatively) by that annual factor does indeed become the final *EPS*, $1.27.

$EPS_0\ (1+g)^n = EPS_n$
$\$1.47(0.9839^9) = \$1.27$, the final EPS

If you accept the arithmetic mean for the norm, you will think that EPS growth has been skyrocketing—nothing could be farther from the truth.

A side trip through the garden of logarithms. By definition, the standard deviation measures a data set's dispersion around its *arithmetic* mean. But we require a standard deviation around its *geometric* mean. Are we out of luck? Not at all! We can find the geometric standard deviation using logarithms. If you are logarithmically challenged, here's a five-paragraph mini-course.

Substituting exponential numbers for ordinary numbers. Any given positive number can be expressed as an exponential power of some other positive number. The "other" number is called a *base* and the required exponent is called the *logarithm* (or *log*) of the given number. For example, 1.995262315 can be expressed as $10^{.3}$ because $10^{.3}$ equals 1.995262315. Ten is the base, and its exponent .3 is the log of 1.995262315. (You can find a number's base-ten logarithm using the LOG function of a spreadsheet or scientific calculator.)

Interpreting exponents. The meaning of the decimal exponent .3 will be clearer if you convert it to its fractional equivalent 3/10. The expression $10^{3/10}$ tells us: "Raise ten to the third power" (10^3 = 10*10*10 = 1000) "and take the tenth root of the result" ($1000^{1/10}$ = 1.995262315). Alternatively and equivalently, $10^{3/10}$ tells us: "Take the tenth root of ten" ($10^{1/10}$ = 1.258925412) "and raise the result to the third power" (1.258925412^3 = 1.995262315). Either way, $10^{.3}$ equals 1.995262315, so the base-ten logarithm of 1.995262315 is .3.

A logarithm is negative when the positive number it represents is less than 1. For example, the log of .501187233 is –0.3. The negative sign in the exponent of $10^{-.3}$ indicates the *reciprocal* of $10^{.3}$, which is $10^{.3}$ divided into 1. Otherwise, the exponent has the same meaning as above. Thus

$10^{-.3}$
$= 1/10^{.3}$
$= 1/1.995262315$
$= .501187233$

Substituting a log sum for a factor product. The product of two or more like-based numbers is the sum of their exponents applied to that base. For example, $(10^{.0406})(10^{.0481}) = 10^{.0406 + .0481} = 10^{.0887}$. Therefore, we can multiply ordinary positive numbers by: (1) adding their corresponding logarithms (i.e., their base-ten exponents) and (2) converting the sum back into an ordinary number, like this.

$(1.097994084)(1.117120445)$
$= (10^{.0406})(10^{.0481})$
$= 10^{.0406 + .0481}$
$= 10^{.0887}$
$= 1.226591639$

The product 1.226591639 is the *antilog* of .0887 (i.e., ten raised to the power of the logarithm .0887).

Substituting a log quotient for a product root. The n^{th} root of an exponential power is the exponent divided by *n* and applied to the same base. For example, the ninth root of $10^{.0774}$ is $10^{.0774/9} = 10^{.0086} = 1.02$. Thus, we can find *the n^{th} root of a product* by adding its factors' logs, dividing the sum by *n*, and converting the quotient to an ordinary number—as here, where we find the square root of a two-factor product.

$[(1.097994084)(1.117120445)]^{1/2}$
$= [(10^{.0406})(10^{.0481})]^{1/2}$
$= 10^{(.0406 + .0481) / 2}$
$= 10^{.0887 / 2}$
$= 10^{.04435}$
$= 1.107515997$, the product's square root (the antilog of .04435)

Back to business. Taken in combination, these facts let us use logarithms to compute the geometric mean and the geometric standard deviation of a set of growth factors.

- First, we add the logs of the *n* growth factors and divide the sum by *n* to get the logs' *arithmetic* mean. The arithmetic mean of the logs is the log of the growth factors' *geometric* mean.

- Then, we find the logs' *arithmetic* standard deviation. The arithmetic standard deviation of the logs is the log of the growth factors' *geometric* standard deviation.

- Finally, we re-express the logs' arithmetic mean and arithmetic standard deviation as ordinary numbers (i.e., we take their antilogs) and arrive at the factors' geometric mean and geometric standard deviation. In summary:

    - The antilog of the logs' arithmetic mean is the geometric mean of the growth factors.

      $10^{\text{arithmetic mean of the factors' logs}}$ = geometric mean of the factors

- The antilog of the logs' arithmetic standard deviation is the geometric standard deviation of the growth factors.

  $10^{\text{arithmetic standard deviation of the factors' logs}}$ = geometric standard deviation of the factors

Applying the procedure. Here again are our bank's previously calculated growth factors, *1+g*, along with their respective base-ten logarithms, *LOG (1+g)*.

1+g	LOG (1+g)
1.0978	.04052
1.1174	.04821
1.1341	.05465
0.9712	-.01269
1.1513	.06119
1.2286	.08941
1.0930	.03862
0.9915	-.00370
1.0150	.00647
LOGSUM	.32268

The nine logs have a sum of .32268, an arithmetic mean of .03585 (their sum divided by nine), and an arithmetic standard deviation of .03322 (computed using a spreadsheet or scientific calculator).

The geometric mean of the factors is *ten to the power of the logs' arithmetic mean*—i.e., $10^{.03585}$, which equals 1.0861.

The geometric standard deviation of the factors is *ten to the power of the logs' arithmetic standard deviation*—i.e., $10^{.03322}$, which equals 1.0795.

Final pitfalls. Even the armor of mathematical precision can't safeguard us from two remaining dangers.

1. The danger that the norms reflect a skewed data set. Ideally, the EPS data set should encompasses a complete business cycle. A data set that contains a partial cycle will likely be biased, over-weighting either the high or the low growth factors.

2. The danger that the norms reflect the bygone state of a changing business. A business is subject to a variety of micro- and macroeconomic forces. The norms of EPS growth are byproducts of these forces. When forces are stable, norms endure—like the wave pattern at the confluence of merging streams. When forces are in flux, the old norms give way to the new. Thus we must be vigilant for currents of change. The predictive power of our calculated norms is determined by the resilience of the forces that produced them.

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