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How to Name a Goat:
Simplifying the "Monty Hall" Problem

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The "Monty Hall" Problem concerns a situation faced by contestants on the TV game show *Let's Make a Deal*. The host, Monty Hall, shows a contestant three closed doors, explaining to her that one hides a new car and the other two hide goats. He then asks her to choose a door and claim her prize.

Having little interest in farm animals, the contestant selects a door suspected to hide the car. Before Monty opens it, however, he opens a different door and reveals a goat. He then gives the contestant a chance to change her mind; to switch from *her* unopened door to the *other* unopened door.

Should she do it? Intuitively, the two doors would seem equally likely to hide the car, in which case switching wouldn't improve her odds. But, curiously, the chances are 2-to-1 that the car is behind the other unopened door. Switching doors will, on average, yield more cars (and fewer goats).

How to think about the probabilities. We tend to frame game problems in terms of *correct* choices. But this one is better framed in terms of *incorrect* choices. Once we realize that the contestant's first choice for the "car" door is likely to be wrong, the problem's probabilities become easier to understand. Here's how they work.

At the outset, contestants are ignorant of the doors' assigned prizes. Their ignorance randomizes the selection process, making each of the three doors an equally probable choice. Since there are two "goat" doors but only one "car" door, two-thirds of the contestants will randomly choose one of the "goat" doors. On these occasions, Monty is *certain* to open the other "goat" door (to avoid displaying the car). When a contestant has chosen one "goat" door and Monty has opened the other "goat" door, the remaining door is *certain* to be the "car" door. Multiplying the individual probabilities gives the joint probability that the car will be behind the remaining door: (2/3)(certainty)(certainty) = (2/3)(1)(1) = 2/3. Two-thirds of the time, switching doors will produce a car.

The other third of the time, switching doors will produce a goat. This happens when the contestant's first choice is the "car" door—a probability of 1/3. Then Monty is *certain* to open one of the "goat" doors and the remaining door is *certain* to be the other "goat" door: (1/3)(certainty)(certainty) = (1/3)(1)(1) = 1/3.

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I have read several explanations of the Monty Hall Problem, but none organized quite like this one. All end in the same probabilities, of course. But some are harder to follow. To find out why, I compared their details.

Overloaded attention. Our attention is not boundless. We can deal simultaneously with only a limited number of things. When the details of an explanation exceed that limit, some drop from awareness. Our consciousness then lacks the complete pattern of facts upon which the conclusion rests, and we fail to grasp its logic.

The number of facts needed to explain the "Monty Hall" Problem depends on something very simple—how the goats and their doors are classified and named. Describe the goats as we did above and the facts are few; the explanation is concise, and its conclusion is obvious. Describe the goats in a different way, and the necessarily larger number can fog our attention and derail our understanding.

Absolute names. Labored explanations of the Monty Hall Problem rely on absolute names—like "goat A" and "goat B"— to distinguish the goats. But absolute names lead to superfluous details. Here's why.

As we have seen, the game involves a three-door sequence: first the contestant's door, then Monty's door, and finally the remaining door. Two of the doors hide goats, and one goat is always behind Monty's door. So, the goats occur in succession: either behind the contestant's door and Monty's door, or behind Monty's door and the remaining door. In the first instance, the three-door sequence will be "goat, goat, car"; in the second, "car, goat, goat."

Within these door sequences, the order of the specific goats doesn't matter. One goat is as good as another. There will be a car behind the remaining door whether the sequence is "goat A, goat B, car" or "goat B, goat A, car." Likewise, there will be a goat behind the remaining door whether the sequence is "car, goat A, goat B" or "car, goat B, goat A." The prize behind the remaining door is simply the converse of the prize behind the contestant's door.

Absolute names tend to obscure the goats' interchangeability and weaken the perception of class likeness. They induce a linguistic hypnosis. They encourage the false assumption that each permutation of the goats is structurally unique—that the succession "goat A, goat B" is a path *functionally* different from the succession "goat B, goat A."

To differentiate these *functionally-irrelevant* permutations is to double the number of probability paths. And because each path requires a separate accounting, the number of details in the explanation must grow accordingly.

Relative names. A simpler explanation avoids this verbal duping. It presents the goats as members of a single *class* of prizes and gives them relative names—"one of the goats" and "the other goat"— variably assigned according to the order in which they are selected. The goat whose door is first chosen (whether by the contestant or Monty) becomes "one of the goats," and the goat whose door is passed over becomes "the other goat."

Relative names confirm the goats' interchangeability and highlight the game's form, which is reducible to just two door-sequences: "goat, goat, car" (when the contestant chooses a "goat" door) and "car, goat, goat" (when the contestant chooses the "car" door).

A visual comparison. These two door-sequences and their probabilities tell the whole story. They are depicted in Table 1. The more cumbersome "absolute-name" framework needs four door-sequences to explain the game: "goat A, goat B, car," "goat B, goat A, car," "car, goat A, goat B," and "car, goat B, goat A." They and their probabilities are depicted in Table 2. A cursory glance at the tables makes it obvious that the first has many fewer details than the second. Thus, it is more easily explained.

DOOR	PRIZE (Its probability)		PRIZE (Its probability)	
Contestant's door	one of the goats	(2/3)	car	(1/3)
Monty's door	the other goat	(1)	one of the goats	(1)
Remaining door	car	(1)	the other goat	(1)
PRODUCT OF THE INDIVIDUAL PROBABILITIES	Sequence 1: Joint probability that a car is behind the remaining door = 2/3		Sequence 2: Joint probability that a goat is behind the remaining door = 1/3	

Table 1. The Monty Hall Problem modeled with relative names

DOOR	PRIZE (Its probability)		PRIZE (Its probability)		PRIZE (Its probability)		PRIZE (Its probability)	
Contestant's door	goat A	(1/3)	goat B	(1/3)	car (1/3)			
Monty's door	goat B	(1)	goat A	(1)	goat A	(1/2)	goat B	(1/2)
Remaining door	car	(1)	car	(1)	goat B	(1)	goat A	(1)
PRODUCT OF THE INDIVIDUAL PROBABILITIES	Sequence 1: Joint probability that a car is behind the remaining door = 1/3		Sequence 2: Joint probability that a car is behind the remaining door = 1/3		Sequence 3: Joint probability that a goat is behind the remaining door = 1/6		Sequence 4: Joint probability that a goat is behind the remaining door = 1/6	
SUM OF THE PROBABILITIES OF LIKE OUTCOMES	Sum of the probabilities that a car is behind the remaining door = 1/3 + 1/3 = 2/3				Sum of the probabilities that a goat is behind the remaining door = 1/6 + 1/6 = 1/3			

Table 2. The Monty Hall Problem modeled with absolute names

Language and perception. The Monty Hall Problem shows the power of language to skew perception and impede understanding. Faulty labels induce muddled thinking. Many intractable problems have yielded once I realized that I had misnamed some important element. For analytical clarity, a thing's name should agree with its structural function.

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